Estimating Output Gaps for the Portuguese Economy:

The Production Function Approach

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Comments welcome

This paper presents estimates of potential output and the output gap for the Portuguese economy, based on an aggregate production function. Recent work in this area by the European Commission is critically examined, and some improvements are suggested. The most prominent of these is the rejection of the Cobb-Douglas functional form and the adoption of a CES specification instead, on grounds of formal econometric tests. The ensuing output gaps are not supportive of the view that the late nineties were a time of economic overheating.

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1. Introduction

The output gap is a widely used concept in applied macroeconomics and economic policy analysis, and has been given an even bigger prominence by the current European fiscal policy framework. Though the Stability and Growth Pact (SGP) deficit ceilings are defined in terms of actual budget figures, rather than cyclically-adjusted ones, the latter undoubtedly play an important role in the overall assessment of fiscal stances and targets, updated every year in the national Stability Programmes. In turn, computing cyclically-adjusted (or structural) deficits requires an estimate of the output gap.

In this context, the European Commission (EC) has recently decided to change its method of computing potential output (and thus output gaps) from the Hodrick-Prescott (HP) filter to an approach based on an aggregate production function (PF). The main advantage of the PF approach lies in its greater economic interpretability. Though refinements of this new method are still under way, its main features are already well established (European Commission, 2002a), and ready to be used in the assessment of the next round of Stability Programmes.

There is nowadays a huge variety of methods to compute output gaps, of which the approaches mentioned above are no more than two examples (see e.g. Cerra and Saxena, 2000, for an overview of such methods). However, given the Commission's pivotal role in the matters related to the application of the SGP, it is arguable that the PF approach has gained prominence (from a policy-oriented perspective) in the wake of the EC's decision to adopt it.

In this paper, we therefore restrict attention to the estimation of potential output according to an aggregate production function. We start by outlining the Commission's method, reviewing its results and identifying its weaknesses (section 2). This leads us to propose some improvements, which is the object of section 3. We resort to different data, reflecting state of the art statistical information from Portuguese national accounts. The Cobb-Douglas functional form assumed by the Commission is formally tested by Johansen's cointegration methods, and is rejected in favour of a more general constant-elasticity-of-substitution (CES) specification. We also propose a number of refinements in matters of econometric estimation and parameter calibration. The final section offers some concluding remarks, and suggests avenues for further research.

2. The European Commission methodology

In this section the method for computing potential output recently adopted by the European Commission is described in detail[1]. We present its results, and also explore its implications in terms of growth accounting and output gap decomposition. Finally, we point out what we think its main limitations are.

2.1 A description

The EC specifies for each member country a Cobb-Douglas aggregate production function with constant returns to scale. The production factors considered are capital (K) and labour (L), both defined for the whole economy[2], and output (Y) is also affected by the level of technology or total factor productivity (A). The labour share in income is calibrated at 0.65 for every country. Therefore:

$$Y_t = A_t K_t^{0.35} L_t^{0.65} \qquad (1)$$

Potential output (Y^*) is obtained by considering total factor productivity and labour at their trend levels (denoted by asterisks), thus removing cyclical components from these two variables:

$$Y_t^* = A_t^* K_t^{0.35} L_t^{*0.65} \qquad (2)$$

The Commission uses annual data from the AMECO database[3], from 1960 to 2003 (these are shown in Table A.1 of the appendix from 1977 onwards). As far as potential

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1 We base our description on European Commission (2002a, 2002c). The former reference includes (in Annex 2) the RATS programme used by the Commission services to compute output gaps, which has been made available to the Portuguese Ministry of Finance together with the associated statistical data. The latter paper is concerned with NAIRU estimation, and its dataset and Kalman Filter software (Planas and Rossi, 2002) have also been made available. We were therefore in a position to replicate the EC results. The interested reader should notice, however, that the Commission services were, at the time of writing, still in the process of refining their methodology. At the time of reading, the latter may therefore have changed somewhat.

2 And not just for the business sector, as in e.g. Giorno *et al.* (1995).

3 The required AMECO (Annual Macro Economic Data Base of DG ECFIN) series are:

output computations are concerned, historical series (up to 2001) and forecasts (2002 and 2003) are handled in a similar way. Potential output is estimated until 2006, but the method for the three out of sample years (2004-2006) is somewhat different, as explained below.

To derive trend total factor productivity the HP filter is used, here and elsewhere with the standard choice (for annual data) of $\lambda = 100$. One starts by generating a series for $\ln(A_t)$ through equation (1). To address the well-known end-point bias, the series for $\ln(A_t)$ is extrapolated until 2008 by a simple autoregressive (AR) process, and only afterwards is the HP filter applied. For each country the AR model has order 3, a constant and a time trend, and is estimated over the 1975-2003 sample. Filtering runs from 1965 to 2008[4].

The actual labour input corresponds to the labour force minus unemployment, and the labour force is in turn defined as the product of the population of working age (aged 15 to 64, *POP*) by the participation rate (*PART*). Hence

$$L_t = POP_t.PART_t(1-u_t), \qquad (3)$$

where u_t is the unemployment rate. Trend or potential employment is instead given by

$$L_t^* = POP_t.PART_t^*(1-u_t^*), \qquad (4)$$

where $PART^*$ is the trend participation rate, and u^* is the NAIRU estimate.

The NAIRU is obtained through a Kalman filter approach, summarised in the following subsection. The trend participation rate results from the HP filter. A series for *PART* is generated using equation (3), and filtered in the 1965-2003 sample. The smoothed

- 1.1.0.0.OVGD (GDP at constant 1995 market prices);
- 1.1.0.0.OIGT (gross fixed capital formation at constant 1995 prices);
- 1.0.0.0.OKND (net capital stock at constant 1995 prices);
- 1.0.0.0.NECD (total civilian employment);
- 1.0.0.0.ZUTN (unemployment rate, Eurostat);
- 1.0.0.0.NPAN (population aged 15 to 64);

[4] It has been decided afterwards to rule out negative growth for A^* (see European Commission, 2002b). This was not a problem for Portugal, but affected especially Spain.

participation rate is then extrapolated until 2006 by an AR(3) model, with a constant and a time trend, estimated over 1990-2003.

To derive potential output beyond 2003 a slightly modified approach is followed. Estimates of A^* and of $PART^*$ have already been obtained (see above). Potential employment continues to be given by equation (4). The working age population used to be extrapolated by an AR model, but the Commission services have decided to use Eurostat projections instead (see European Commission, 2002b). As for the NAIRU after 2003, the following simple projection rule, retaining half of the most recent change, is used:

$$u^*_{t+1} = u^*_t + 0.5(u^*_t - u^*_{t-1})\,. \qquad (5)$$

The only remaining task before being able to use equation (2) is to project the capital stock. A first step consists in modelling the investment[5]-to-potential output ratio (IY^*) by means of an AR(4) process with a constant and a time trend. This simple autoregressive model is estimated in the 1975-2003 sample, and used to forecast IY^* in 2004-2006. In these three years, the depreciation rate (*dep*) is set equal to its value in 2003. Then investment (*I*), the capital stock and potential output are jointly determined by the three-equation system formed by (2) and by equations (6) and (7) below:

$$K_t = I_t + (1 - dep_t)K_{t-1} \qquad (6)$$

$$I_t = IY^*_t . Y^*_t \qquad (7)$$

Until 2003, the output gap (*YGAP*) is given by Y/Y^* - 1. From 2004 onwards no forecast of *Y* is available. It is *assumed* that the economies reach full employment (*YGAP* = 0) by 2006, the output gap in 2004 being two thirds of its 2003 value, and one third in 2005. Having obtained/assumed values for Y^* and for *YGAP*, the ensuing *Y* figures can of course be computed.

[5] More precisely, the gross fixed capital formation.

2.2 NAIRU estimation

The European Commission (2002c) uses an unobserved components model estimated by the Kalman filter to obtain time-varying values of the NAIRU. The method is summarised below.

The NAIRU is estimated as the trend component (T_t) of the unemployment rate, which also includes a cyclical component (C_t):

$$u_t = T_t + C_t \qquad (8)$$

To identify C_t a Phillips curve relationship is used, whose general expression can be written as

$$\Delta\pi_t^w = \mu + A(L)\Delta\pi_t^w + \gamma\Delta u_{t-1} + B(L)C_t + \delta X_t + C(L)\varepsilon_t, \qquad (9)$$

where *A(L)*, *B(L)* and *C(L)* are lag polynomials, the left-hand side variable is the first difference of wage inflation and X_t is a vector of exogenous variables. This formulation can be regarded as inspired by Gordon's (1997) 'triangle-model' of inflation, whereby the latter depends on inertia, demand and supply (vector X_t contains supply-side determinants, such as terms of trade shocks). At the time of writing (July 2002), the Commission services have specialised the above equation in the case of Portugal to

$$\Delta\pi_t^w = \gamma\Delta u_{t-1} + \beta C_t + \delta\Delta TOT_{t-1} + (1 + \theta_1 L + \theta_2 L^2 + \theta_3 L^3)\varepsilon_t, \qquad (10)$$

where TOT_t captures terms of trade shocks (it is defined as the growth rate of the ratio between the deflators of GDP and consumption), and ε_t is a normally distributed white noise (like the shocks v_t, z_t and ξ_t defined below). Besides being economically interpretable, the cyclical component should also have suitable statistical properties. These are stationarity, a sample mean of zero and an AR(2) specification:

$$C_t = \phi_1 C_{t-1} + \phi_2 C_{t-2} + v_t, \qquad (11)$$

with appropriate stationarity restrictions imposed on the coefficients. The disturbance v_t could in principle be correlated with ε_t, but the inclusion of C_t in equation (10) imposes the restriction that such correlation be zero.

No economic theory is used to identify the trend component (i.e., the NAIRU), which is specified as a second order random walk:

$$T_t = \mu_t + T_{t-1} + z_t , \mu_t = \mu_{t-1} + \xi_t \quad (12)$$

As is standard in the literature, the variances of the shock terms (especially z_t) are constrained to lie within a certain range, in order to ensure an adequate degree of smoothness for the NAIRU series. The most recent estimates of the latter are given in Table A.1[6].

2.3 Results: output gaps and growth accounting

Table 1 presents results from 1977 onwards. The first column shows GDP growth, the second potential output growth, and the sixth the output gap (the remaining columns will be referred to below). A discussion of these figures is deferred to section 3, where they will be compared with estimates obtained from a different methodology. We note, however, that Table 1 presents some differences relative to the EC papers (2002a, 2002b): these are due to the use of a more recent series for the NAIRU, as well as to data revisions, since those papers were based on provisional Spring 2002 forecasts, whereas we use their definitive counterparts.

The simplicity of the Cobb-Douglas functional form makes it easy to perform growth accounting exercises. To decompose the growth of potential output into the contributions of labour, capital and technical progress, we take first differences in equation (2), written in logs (lower-case variables henceforth denote the natural logarithm of their upper-case counterparts):

$$\Delta y_t^* = \Delta a_t^* + 0.35\Delta k_t + 0.65\Delta l_t^* \quad (13)$$

[6] The EC methodology is still undergoing refinements, and its most recent estimates differ somewhat from those reported in European Commission (2002a).

The three terms on the right hand side are the abovementioned contributions, and are given in Table 1 (third, fourth and fifth columns, respectively). The overall picture that emerges (with the exception of the late eighties and early nineties) is that of growth mainly propelled by factor accumulation, rather than by technical progress.

Table 1 – Results of the Commission's Method

	WY	WY*	WY*S	WY*K	WY*L	YGAP	TGAP	PGAP	UGAP
1977	5.5	3.5	1.6	1.8	0.0	-0.4	0.7	-0.3	-0.7
1978	2.8	3.0	1.3	1.8	0.0	-0.6	0.7	-0.5	-0.7
1979	5.6	2.9	1.0	1.6	0.2	2.1	2.1	0.3	-0.3
1980	4.6	3.4	0.8	1.9	0.7	3.3	2.6	0.4	0.2
1981	1.6	3.6	0.6	2.2	0.8	1.4	1.9	-1.2	0.6
1982	2.1	3.2	0.5	2.0	0.7	0.4	1.3	-1.7	0.7
1983	-0.2	2.5	0.5	1.7	0.3	-2.2	-4.2	1.6	0.4
1984	-1.9	2.3	0.7	0.9	0.7	-6.3	-6.9	0.5	-0.1
1985	2.8	2.6	0.9	0.8	0.9	-6.1	-5.5	-0.1	-0.5
1986	4.1	2.9	1.2	0.8	0.8	-4.9	-3.5	-0.7	-0.7
1987	6.4	3.9	1.4	1.2	1.3	-2.6	-1.3	-0.8	-0.5
1988	7.5	3.9	1.5	1.5	0.9	0.7	1.3	-0.5	-0.1
1989	6.4	3.3	1.6	1.5	0.2	3.8	3.2	0.2	0.3
1990	4.0	3.3	1.5	1.5	0.3	4.4	2.7	1.0	0.6
1991	4.4	3.7	1.4	1.4	0.8	5.1	2.3	2.0	0.7
1992	1.1	3.5	1.3	1.4	0.7	2.7	2.0	0.2	0.4
1993	-2.0	2.5	1.2	1.1	0.1	-1.8	-1.2	-0.5	-0.1
1994	1.0	2.4	1.2	1.1	0.1	-3.1	-2.3	-0.3	-0.5
1995	4.3	2.6	1.1	1.1	0.4	-1.5	0.0	-0.8	-0.7
1996	3.8	2.5	1.0	1.2	0.3	-0.2	1.2	-0.8	-0.6
1997	3.9	2.8	0.9	1.5	0.4	0.9	1.3	-0.3	-0.2
1998	4.5	3.8	0.8	1.8	1.1	1.6	1.4	-0.2	0.3
1999	3.4	3.2	0.8	1.9	0.5	1.8	1.0	0.1	0.6
2000	3.4	3.4	0.7	2.0	0.7	1.8	0.6	0.5	0.6
2001	1.8	3.9	0.7	1.7	1.5	-0.3	-1.1	0.4	0.4
2002	1.5	3.2	0.8	1.5	0.9	-2.0	-2.1	0.2	-0.1
2003	2.2	3.3	0.8	1.5	1.0	-3.0	-2.4	-0.2	-0.5
2004	3.8	2.7	0.9	1.4	0.4	-2.0			
2005	4.0	2.9	1.0	1.5	0.4	-1.0			
2006	4.2	3.1	1.0	1.5	0.6	0.0			

All values are percentage points. WY is GDP growth, WY* is potential output growth, WY*S, WY*K and WY*L are the contributions of technical progress, capital and labour to the growth of potential output, YGAP is the output gap, and TGAP, PGAP and UGAP are the technology, participation and unemployment gaps. These are not computed after 2003 since we lack projections of the unemployment rate after that year. In 2004-06 WY follows from the results and assumptions for WY* and YGAP (see end of section 2.1).

We can also decompose the output gap into its possible sources, which are three in the current framework:

- a *technology gap* (TGAP) following from the departures of total factor productivity from its smoothed trend;

- a *participation gap* (PGAP) resulting from the difference between actual and smoothed participation rates;

- an *unemployment gap* (UGAP) arising from the deviation of the actual unemployment rate from the NAIRU.

Formally, we subtract equation (2) from equation (1), both in logs:

$$y_t - y_t^* = a_t - a_t^* + 0.65(l_t - l_t^*) \tag{14}$$

Bearing in mind the expressions giving l_t and l_t^* (equations (3) and (4), in logs), and using the approximation $\ln(1-x) \cong -x$ (for small *x*), we write

$$y_t - y_t^* \cong \left[a_t - a_t^*\right] + \left[0.65(part_t - part_t^*)\right] + \left[0.65(u_t^* - u_t)\right], \tag{15}$$

where the terms in square brackets are respectively the technology gap, the participation gap and the unemployment gap. These are given in the three last columns of Table 1. One can see that, according to this decomposition, the technology gap is generally dominant. It is also interesting to remark that the participation gap tends to be somewhat bigger than the unemployment gap, with average absolute values over 1977-2001 of 0.64 against 0.47. Both these gaps are pro-cyclical, displaying correlation coefficients with *YGAP* over 1977-2001 of 0.32 and 0.61, respectively. This pro-cyclicality is not a novel finding, but reinforces other studies (e.g. Bonfim and Neves, 2002).

2.4 Some limitations

The PF approach as implemented by the Commission services offers important advantages over purely statistical methods (e.g. the HP filter), namely in terms of the economic interpretability of results. The previous section is a good example. However, we find some aspects of the methodology debatable.

First, a Cobb-Douglas specification, though appealing on grounds of simplicity and analytical tractability, imposes the strong restriction of a unit elasticity of substitution: we contend that this assumption can and should be tested, rather than imposed.

Second, all countries are treated identically as regards AR specifications and Cobb-Douglas parameter values, which imposes untested uniformity across (possibly) different national structures. As a result, some of the simple AR models for Portugal are misspecified and/or not parsimonious[7].

Third, despite the merits of the AMECO database as a source of readily available, internationally comparable information, some of its series for Portugal differ significantly from recent statistical work done at the *Direcção-Geral de Estudos e Previsão* (DGEP) of the Portuguese Ministry of Finance. We return to this issue in section 3.1.

Fourth, results for potential output after 2003 should be regarded with great caution: with the exception of the population of working age, all the remaining inputs are obtained via simple time series extrapolations, and are therefore surrounded by considerable uncertainty. Likewise, the assumption that output gaps gradually close until 2006 is arbitrary, and together with the estimates for Y^* may imply unrealistic rates of GDP growth.

Finally, the PF approach still relies heavily on HP filtering – especially because this filter is used to smooth total factor productivity, which is the main source of output volatility[8]. We illustrate this point by decomposing the variability of GDP growth. From $\Delta y_t = \Delta a_t + 0.35\Delta k_t + 0.65\Delta l_t$ one can write

$$\mathrm{var}(\Delta y_t) = \mathrm{var}(\Delta a_t) + \mathrm{var}(0.35\Delta k_t + 0.65\Delta l_t) + 2\,\mathrm{cov}(\Delta a_t, 0.35\Delta k_t + 0.65\Delta l_t)\,. \qquad (16)$$

Over 1977 to 2001, this identity reads 5.530 = 4.395 + 1.695 + 2x(-0.280), showing that variability in the growth of the production factors is clearly dominated by variability in the growth of total factor productivity.

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[7] The AR(3) process for a_t presents residual autocorrelation according to the Breusch/Godfrey test, and the coefficient on the 3rd lag is not significant. The AR(3) process for the smoothed participation rate and the AR(4) process for IY^* also present problems of residual autocorrelation, and in the latter model the coefficients on the 3rd and 4th lags are not significant. Related issues where we believe there is room for improvement are the use of different subsamples for different AR models, and the fact that PART is first filtered by HP, and only afterwards extrapolated forward, rather than the other way round.

[8] Not to mention the use of the HP filter to smooth the participation rate as well.

3. An alternative methodology

In developing an alternative methodology, we try to address some the less satisfactory features of the EC computations, pointed out in the previous subsection. At the same time, for the reasons discussed in the introduction, we wish to remain within the PF approach, and to ease comparability with the Commission's work. Therefore, we limit ourselves to a CES specification (of which the Cobb-Douglas is a particular case), and to two production factors, labour and (total) capital.

3.1 Data

The change from ESA 79 to ESA 95 induced a break in national accounts series, creating the need to adjust ESA 79 data to make it as compatible as possible with the new system. This work has recently been done by DGEP (2002): they prepared series from 1977 to 1997 that take ESA 95 data (starting in 1995) as a basis and chain backwards using ESA 79 growth rates for previous years. Table A.1 in the appendix contains such series for GDP, employment and gross fixed capital formation. From 1999 onwards figures are estimates or projections.

Historical data and projections for the working age population have been obtained from the *Instituto Nacional de Estatística* (INE). The source for the unemployment rate is also INE (*Inquérito ao Emprego*). As in the AMECO database, the net capital stock series has been constructed by positing an initial value for 1960 (corresponding to a K/Y ratio of 3) and cumulating gross fixed capital formation minus capital consumption afterwards (from 1999 onwards a simple ARIMA model was used to forecast the rate of depreciation).

Though we leave an in-depth analysis of the NAIRU estimation as a topic for further work, it would be wrong to simply insert the Commission's NAIRU series into our computations. The reason is that the NAIRU is the trend component of the unemployment rate, and there are differences (though slight) in this rate between the AMECO and the DGEP datasets: the observed rates being different, assuming that their trend component is the same would be an oversimplification. We have then applied the Commission's method (section 2.2) and the associated Kalman filter software to the

DGEP's series for the unemployment rate, wage inflation and the terms of trade variable.

Table A.1 compares the AMECO and DGEP datasets in their common period (1977-2003). It can be seen that there are non-trivial differences, especially in the early years of the sample and as regards employment.

Table 2 – The Commission's Method applied to the DGEP dataset

	WY	WY*	WY*S	WY*K	WY*L	YGAP	TGAP	PGAP	UGAP
1977						-1.8	-1.3	0.3	-0.7
1978	3.4	2.7	1.3	1.8	-0.4	-1.1	0.0	-0.4	-0.7
1979	6.1	2.5	1.3	1.6	-0.4	2.3	1.7	1.0	-0.3
1980	4.8	3.1	1.3	1.9	-0.1	4.1	3.4	0.3	0.2
1981	1.3	2.4	1.3	2.2	-1.0	2.9	0.7	1.4	0.7
1982	2.1	4.0	1.3	2.0	0.7	1.0	0.7	-0.2	0.4
1983	-0.2	2.9	1.3	1.7	-0.1	-2.1	-1.6	-0.4	-0.1
1984	-1.8	2.3	1.4	0.9	-0.1	-6.0	-4.8	-0.8	-0.5
1985	3.0	2.2	1.6	0.7	-0.1	-5.3	-4.2	-0.4	-0.8
1986	4.1	2.3	1.7	0.8	-0.2	-3.6	-0.9	-1.9	-0.8
1987	5.9	3.4	1.8	1.2	0.4	-1.2	0.5	-1.2	-0.4
1988	5.5	3.4	1.8	1.3	0.3	0.9	1.3	-0.6	0.1
1989	5.4	2.9	1.7	1.3	-0.1	3.3	2.4	0.3	0.6
1990	4.8	3.0	1.6	1.4	-0.1	5.1	2.9	1.1	0.9
1991	2.4	4.2	1.5	1.4	1.2	3.3	0.7	1.8	0.9
1992	1.9	3.4	1.4	1.3	0.6	1.8	0.9	0.4	0.5
1993	-1.4	2.3	1.3	1.0	-0.1	-1.8	-1.5	-0.3	-0.1
1994	2.5	2.4	1.2	1.0	0.1	-1.8	-0.6	-0.5	-0.6
1995	2.9	2.6	1.2	0.9	0.5	-1.5	0.3	-0.8	-0.9
1996	3.5	2.5	1.1	1.0	0.3	-0.4	0.6	-0.1	-0.9
1997	3.9	3.0	1.0	1.4	0.5	0.5	1.0	0.0	-0.5
1998	4.5	3.8	1.0	1.7	1.1	1.3	1.4	-0.2	0.1
1999	3.5	3.1	1.0	1.7	0.4	1.7	1.1	-0.1	0.6
2000	3.5	3.3	1.0	1.7	0.6	1.9	0.7	0.2	0.9
2001	1.9	3.4	1.0	1.6	0.8	0.4	-1.0	0.5	0.8
2002	1.0	3.4	1.1	1.4	0.9	-2.0	-2.7	0.3	0.4
2003	1.7	3.2	1.3	1.3	0.6	-3.3	-3.3	0.0	-0.1
2004	4.5	3.3	1.4	1.4	0.5	-2.2			
2005	4.8	3.7	1.6	1.5	0.5	-1.1			
2006	5.1	3.9	1.7	1.6	0.5	0.0			

See the notes under Table 1.

To study whether these differences have an important impact on the estimates of the output gap, Table 2 shows the results of applying the Commission's methodology to the DGEP dataset[9]. Output gaps are roughly similar, though the cyclical deterioration in 2002 and 2003 is somewhat bigger. On the other hand, the growth accounting exercise

[9] The only difference relative to the methodology described in section 2.1 is that the data only starts in 1977: whenever a longer sample was used by the EC (e.g. the 1975-2003 sample used to estimate the AR(3) model for a_t), its starting date was changed to 1977. Since the DGEP series for POP finishes in

yields quite a different picture, with technical progress carrying more weight than in Table 1. The pro-cyclicality of the participation rate is confirmed and even reinforced, with a 0.72 correlation between *PGAP* and *YGAP* over 1977-2001. Needless to say, the criticisms made in section 2.4 continue to apply (notice, for instance, the very strong implied GDP growth in 2004-2006).

3.2 A CES Production Function

With the exception of Marques (1990), previous work on the estimation of Portuguese potential output through the PF approach has assumed a Cobb-Douglas functional form (Botas *et al.*, 1998; Pinheiro, 1998). Our main purpose in this subsection is to test in a formal way whether a Cobb-Douglas specification over labour and total capital is a valid assumption. In the more general CES case, we test the parameter restrictions implied by Cobb-Douglas, and reject them unequivocally. We then calibrate the retained CES production function.

Econometric tests

The CES production function with constant returns to scale is written as

$$Y_t = \gamma\left[\delta(K_t)^\rho + (1-\delta)(L_t e^{\eta t})^\rho\right]^{1/\rho}, \qquad (17)$$

where technical progress takes place at rate η, while the elasticity of substitution σ equals $1/(1-\rho)$. We assume technical progress to be labour-augmenting (Harrod-neutral), as in much of the literature[10].

Since the CES function is non-linear, even in logarithmic form, parameter values are usually obtained through the estimation of factor demands (e.g. Barrell and Pain, 1997;

2005, a projection for 2006 was made assuming the same growth rate of the Commission series. We have used the Commission's NAIRU estimates.

[10] In the neoclassical growth model with constant rates of technical progress, Harrod neutrality is theoretically appealing, since it is consistent with the existence of a steady state, while Hicks or Solow neutrality are not (see Barro and Sala-i-Martin, 1995, pp. 54-55).

Dimitz, 2001). The labour demand equation, derived from equating the marginal product of labour to the real wage[11], can be rearranged as

$$\ln(Y_t / L_t) = \sigma \ln(W_t / P_t) + (1-\sigma)\eta t \, , \tag{18}$$

where constant terms are omitted. The logs of average labour productivity and the real wage are usually non-stationary variables (as we confirm below), and so the equation above should be regarded in econometric terms as a cointegrating relation.

The Cobb-Douglas function has a unit elasticity of substitution, implying that the coefficient of the (log) real wage is one, and that of the time trend becomes zero. However, in the presence of integrated variables these two restrictions cannot be tested through a standard F-test. Instead, Johansen's analysis of cointegrated vector autoregressions (VARs) offers us the appropriate tests (see e.g. Johansen, 1995, for a textbook presentation).

Our econometric approach has therefore proceeded in four steps. First, we have studied the order of integration of the variables. Second, a bivariate VAR in average labour productivity and the real wage has been estimated. Third, we have tested for cointegration using the maximal eigenvalue (λ_{max}) and trace eigenvalue (λ_{trace}) statistics. Fourth, assuming that cointegration exists, we have tested (by maximum likelihood) restrictions on the coefficients of the cointegrating vector.

An effort has been made to match as closely as possible statistical data to the theoretical concepts involved. Our output variable is GDP at factor cost, and not at market prices, since the former is more appropriate to define productivity from a firm's viewpoint. When computing labour productivity we have used total employment (comprising both employees and the self-employed), but to calculate average wages remunerations (including employers' social contributions) have been divided by employees only, since national accounts record in the gross operating surplus the income earned by the self-

[11] Perfect competition is not an essential assumption here: a constant mark-up may exist. One should also note that a similar labour demand equation would follow from Hicks-neutral technical progress.

employed[12]. Our sample runs from 1978 to 1997, the period for which national accounts data are available on a comparable basis (see DGEP, 2002)[13].

Table 3 summarises the Dickey-Fuller (DF) and Augmented Dickey-Fuller (ADF) tests for unit roots. Though the small size of our sample complicates statistical inference, here and elsewhere, the tests point to both variables (labour productivity and the real wage) being I(1), and we treat them as such in the remainder.

Table 3 – Unit root tests

	$\ln(Y_t / L_t)$	$\ln(W_t / P_t)$
H_0: I(1)	-2.65	-1.97
H_0: I(2)	-4.05***	-2.75*

The table reports DF/ADF test statistics. To test the null of I(1) we have used ADF(1) tests with a constant and a time trend, and to test the null of I(2) we have used DF tests with a constant only. Our sample is 1977-1997. Asterisks *, ** and *** denote rejection at 10%, 5% and 1% critical values, respectively, and the same applies in the rest of this paper.

A first-order VAR, including an intercept and a time trend[14], was found to be statistically acceptable, and used to test for cointegration. Table 4 summarises the test statistics computed to determine the cointegration rank r[15].

Table 4 – VAR cointegration tests

	λ_{max}	λ_{max}^{a}	95%	90%	λ_{trace}	λ_{trace}^{a}	95%	90%
H_0: r = 0	19.56**	17.5*	19.0	16.9	23.28*	20.83	25.3	22.8
H_0: r ≤1	3.726	3.334	12.3	10.5	3.726	3.334	12.3	10.5

Columns '95%' and '90%' contain critical values, taken from Table 2* of Osterwald-Lenum (1992). Superscript 'a' denotes a degrees-of-freedom adjustment.

There is some evidence to reject $r = 0$, i.e. to accept the existence of cointegration, and we assume in the remainder that cointegration does exist. Imposing r = 1 yields the cointegrating vector

[12] Data limitations therefore prevent us from defining an average wage encompassing all labour. Yet another limitation when computing real wages follows from the fact that a deflator for GDP at factor cost is not available: we have used that of GDP at market prices instead.
[13] We have excluded 1977 from the sample, as it was found to be an outlier.
[14] The time trend has been restricted to lie in the cointegration space, so as to prevent a quadratic deterministic trend in the level of variables. The estimation and ensuing hypothesis testing was performed in PcFiml 9.0 (Doornik and Hendry, 1997).
[15] In a bivariate integrated VAR the existence of cointegration is of course equivalent to $r = 1$.

$$\ln(Y_t / L_t) = 0.27965 \ln(W_t / P_t) + 0.016313t, \quad (19)$$

implying $\sigma = 0.27965$ and $\eta = 0.022646$, parameter values which are discussed below. The two restrictions imposed by a Cobb-Douglas specification can then be tested, and are clearly rejected: $\chi^2(2) = 13.119$, with a P-value of 0.0014[16].

Since the Cobb Douglas function is log-linear, one may also apply an Engle-Granger (EG) test to its residuals, though this possibility is not available for the more general CES formulation. For completeness, we have considered both the AMECO and the DGEP series, and both estimated and calibrated labour shares. The null of a unit root in the residuals could never be rejected, as documented in table 5.

Table 5 – Unit root tests in the Cobb-Douglas case

Sample	AMECO, 1960-2001	AMECO, 1960-2001	DGEP, 1977-2001	DGEP, 1977-2001	DGEP, 1977-2001
Labour share	0.92, est.	0.65, cal.	0.86, est.	0.65, cal.	0.73, cal.
Test	EG(3,C)	ADF(2,CT)	EG(C)	ADF(1,CT)	DF(CT)
H_0: I(1)	-1.63 [0.756]	-2.46 [0.355]	-1.97 [0.572]	-2.40[0.368]	-1.67 [0.727]

The second row indicates whether the labour coefficient α has been estimated or calibrated. In the latter case, residuals are computed as $y - \alpha l - (1- \alpha)k$; in the former, we estimate by OLS a log-linear Cobb-Douglas function with an intercept, a trend and constant returns to scale. Calibration either adopts the Commission's figure of 0.65 (recall section 2.1) or the alternative labour share of 0.73, explained below. The third row states the test we use: figures give the number of lags, C and T indicate whether a constant or a trend have been included. The final row reports test statistics, and P-values in square brackets.

We conclude from the above econometric analysis that a Cobb-Douglas production function is not supported by the data. If we retain the assumption of a constant elasticity of substitution, the evidence suggests that it should be set at around 0.3.

Calibration

The CES production function has four parameters to calibrate: γ, δ, η and ρ (or, equivalently, σ). Values for ρ and η follow from the econometric analysis above. Our

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[16] It has also been tested whether the coefficient of time in the cointegrating vector was zero, leaving σ unrestricted: we obtained $\chi^2(1) = 13.092$, with a P-value of 0.0003. This can be interpreted as a rejection of technical progress being exclusively of the Solow neutral kind.

estimated elasticity of substitution is much lower than unity (0.28), though not very different from some estimates for other countries[17]. Interestingly, we get almost the same value as Marques (1990), who obtained a figure of 0.31 with a different sample and estimation approach. Our estimate for the rate of labour-augmenting technical progress (2.3%) is similar to those commonly found[18].

On the assumption that labour and capital are remunerated according to their marginal productivities, one can set output elasticities *w.r.t.* each of the production factors equal to the respective shares in income. Some algebra shows that

$$\frac{1-\delta}{\delta} = \frac{W_t L_t / (P_t Y_t)}{1 - W_t L_t / (P_t Y_t)} \left(\frac{L_t e^{\eta t}}{K_t} \right)^{-\rho}, \qquad (20)$$

from which one can calibrate δ. Finally, the scale parameter γ was chosen so as to ensure that the average output gap over the 1977-1997 period coincides with that of Table 1 (–0.2)[19].

To calibrate δ we need to know the labour share in income, which is made difficult by the fact that remunerations refer to employees only (see the previous subsection). Remunerations' weight in GDP at factor cost is approximately 0.56 (1977-1997 average), but this figure understates the labour share, since some of the income earned by the self-employed should be regarded as labour income from an economic point of view. If we assume that in each sector the underlying remuneration of the self-employed equals that of employees[20], the labour share rises to 0.73, which is admittedly high by international standards. We retain this figure (0.73), but perform some sensitivity analysis in the appendix. Table 6 summarises our calibration.

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[17] For instance, Barrell and Pain (1997) find a value of 0.39 for West Germany, and of 0.41 for the UK manufacturing sector.

[18] For instance, in the well-known macroeconometric model NiGEM.

[19] A different criterion could of course be chosen.

[20] Willman (2002) makes a similar assumption for the whole economy, without sectoral disaggregation. We argue that the latter is important, at least in the Portuguese case, since a large proportion of the self-employed work in the low-wage agricultural sector. We use the same eight-sector disaggregation as Direcção-Geral de Estudos e Previsão (2002).

Table 6 – Calibration of the CES production function

	ρ	η	δ	γ
Values	-2.57590	0.022646	0.27471x10^{-4}	0.01280

3.3 Other methodological issues

We estimate potential output by inserting into the CES function the actual capital stock and trend employment:

$$Y_t^* = \gamma\left[\delta(K_t)^\rho + (1-\delta)(L_t^* e^{\eta t})^\rho\right]^{1/\rho} \quad (21)$$

Our treatment of trend or potential employment is similar to that of section 2.1, and only minor refinements are introduced. These concern the participation rate. We find that a simple AR(1) model with intercept but no time trend is an adequate representation of *PART* in the 1977-2001 sample (the years 2002 and 2003 are excluded from the estimation, as they are not historical data). We use this model to generate forecasts from 2004 to 2008, and then apply the HP filter from 1977 to 2008. As regards the NAIRU, the Commission's method has been applied to DGEP's data, as explained in section 3.1.

If one takes the CES function, plugs into it the actual values of *Y*, *K* and *L* (and time), and solves for γ, one will not find in general the constant given in table 6, but rather a time series of values (γ_t) fluctuating around it. Thus one actually has:

$$Y_t = \gamma_t\left[\delta(K_t)^\rho + (1-\delta)(L_t e^{\eta t})^\rho\right]^{1/\rho} \quad (22)$$

The divergence between γ and γ_t can be explained by a host of factors, such as changes in the degree of capacity utilisation or temporary productivity shocks. If one wishes to further smooth potential output, it is possible to filter (by HP) γ_t, denoting the outcome with an asterisk[21]:

[21] We actually work with ln(γ_t), rather than γ_t, Before filtering, ln(γ_t) is extrapolated until 2008 by means of a AR(2) process with a constant, estimated in the 1977-2001 sample.

$$Y_t^* = \gamma_t^* \left[\delta (K_t)^\rho + (1-\delta)(L_t^* e^{\eta t})^\rho \right]^{1/\rho} \tag{23}$$

Thus equations (21) and (23) give two alternative definitions of potential output, and results for both will be presented in the following section. We will only estimate potential output until 2003, for the reasons given in section 2.4. However, if one wishes to go further into the future and has projections for *POP*, nothing prevents the use of a method similar to the Commission's, based on AR forecasts for IY^*. Other possibilities also exist. For instance, one may take a certain path for *Y* (e.g. from the national Stability Programme), model the growth of *I* as a function of the growth of *Y*[22], and thus, 'knowing' *Y*, obtain projections for *I*, *K* and Y^*.

3.4 Results: output gaps and growth accounting

Growth accounting and output gap decomposition have now less straightforward expressions than in the Cobb-Douglas case, but their computation remains feasible.

As for growth accounting, one can approximate the growth rate of Y^* by a linear function of the rates of growth of *K*, *L*, technical progress and γ^*, each multiplied by the respective elasticity. Technical progress grows at rate η, and is multiplied by $\varepsilon^{Y,L}$ (the elasticity of output with respect to labour) due to its labour augmenting nature. The 'elasticity' that multiplies γ^* is unity. Unlike in the Cobb-Douglas case, elasticities are not constant, and we evaluate them at *t-1*. Then:

$$\Delta y_t^* \cong \left[\Delta \ln \gamma_t^* + \varepsilon_{t-1}^{Y,L}\eta\right] + \varepsilon_{t-1}^{Y,K}\Delta k_t + \varepsilon_{t-1}^{Y,L}\Delta l_t^* . \tag{24}$$

The elasticities are given by

$$\varepsilon_{t-1}^{Y,K} = \delta\left(\frac{\gamma_{t-1}^* K_{t-1}}{Y_{t-1}^*}\right)^\rho ; \;\; \varepsilon_{t-1}^{Y,L} = (1-\delta)\left(\frac{\gamma_{t-1}^* L_{t-1}^* e^{\eta(t-1)}}{Y_{t-1}^*}\right)^\rho . \tag{25}$$

22 A simple model with good statistical properties, estimated in the 1977-2001 sample, is given by $\Delta i_t = -0.045 + 0.370\Delta i_{t-1} - 0.260\Delta i_{t-2} + 2.807\Delta y_t$ (recall that lower-case letters denote logs).

The three terms on the right-hand side of (24) are the contributions of technical progress (broadly understood), capital and labour to the growth of potential output[23]. Notice that $\Delta \ln \gamma_t^* \equiv 0$ if one uses equation (21) instead of (23).

Because we model technical progress by a time trend, we cannot speak of a technology gap in the same sense of section 2.3. We can nonetheless define a 'technology gap' (probably better called a capacity gap) as $\ln(\gamma_t) - \ln(\gamma_t^*)$ ($\ln(\gamma_t) - \ln(\gamma)$ in the case of (21)). The remainder of the output gap can be imputed to the difference between actual and potential employment, and we decompose this labour gap into a participation gap and an unemployment gap according to the weights of $part_t - part_t^*$ and $u_t^* - u_t$ in $(part_t - part_t^*) + (u_t^* - u_t)$. Tables 7 and 8 give results.

Table 7 – Results from the CES production function (21)

	WY	WY*	WY*S	WY*K	WY*L	YGAP	TGAP	PGAP	UGAP
1977						-2.2	-1.8	0.3	-0.8
1978	3.4	2.9	1.4	1.9	-0.4	-1.8	-0.7	-0.4	-0.7
1979	6.1	2.7	1.5	1.7	-0.4	1.5	0.9	0.9	-0.3
1980	4.8	3.2	1.5	1.9	-0.1	3.1	2.5	0.4	0.3
1981	1.3	2.4	1.5	2.0	-1.0	2.0	-0.4	1.6	0.8
1982	2.1	4.1	1.6	1.7	0.8	0.0	-0.3	-0.3	0.5
1983	-0.2	2.8	1.6	1.3	-0.1	-3.0	-2.5	-0.4	-0.1
1984	-1.8	2.3	1.7	0.7	-0.1	-6.9	-5.6	-0.8	-0.5
1985	3.0	2.1	1.7	0.6	-0.1	-6.1	-4.8	-0.4	-0.8
1986	4.1	2.0	1.7	0.6	-0.3	-4.1	-1.0	-2.2	-0.9
1987	5.9	3.0	1.7	0.9	0.4	-1.4	0.5	-1.4	-0.5
1988	5.5	3.0	1.7	1.0	0.3	1.0	1.6	-0.7	0.1
1989	5.4	2.5	1.7	1.0	-0.1	3.9	2.9	0.3	0.7
1990	4.8	2.6	1.7	1.0	-0.1	6.1	3.5	1.4	1.1
1991	2.4	4.2	1.7	0.9	1.5	4.3	1.3	2.0	1.0
1992	1.9	3.4	1.7	0.9	0.8	2.8	1.7	0.4	0.6
1993	-1.4	2.4	1.7	0.7	-0.1	-1.0	-0.5	-0.4	-0.1
1994	2.5	2.6	1.7	0.7	0.2	-1.1	0.3	-0.7	-0.8
1995	2.9	3.0	1.7	0.6	0.6	-1.2	1.0	-1.1	-1.1
1996	3.5	2.8	1.7	0.7	0.4	-0.5	0.9	-0.3	-1.1
1997	3.9	3.3	1.7	1.0	0.5	0.2	0.9	-0.1	-0.6
1998	4.5	4.1	1.7	1.1	1.3	0.6	0.8	-0.3	0.1
1999	3.5	3.3	1.7	1.1	0.5	0.8	0.0	-0.1	0.8
2000	3.5	3.5	1.8	1.1	0.6	0.8	-0.7	0.4	1.1
2001	1.9	3.6	1.8	0.9	0.9	-0.9	-2.8	0.9	1.0
2002	1.0	3.5	1.8	0.8	0.9	-3.3	-4.7	0.9	0.5
2003	1.7	3.0	1.8	0.8	0.4	-4.5	-5.2	0.9	-0.2

See the notes under Table 1.

Output gap estimates are broadly in line with those produced by the European Commission, especially in the case of Table 8, due to the treatment of parameter γ

[23] Equation (24), with the coefficients given by (25), can be reached in a more formal way by taking logs

(though in this case we are again giving an important role to the HP filter in our PF approach). Chart 1 shows that the most important divergence between the CES function (21) and the EC gaps takes place from 1993 onwards. The CES function points to a milder cycle in the nineties, both in the 1993/94 crisis and in the expansion towards the end of the decade[24]. In 2001-2003 potential output growth is similar in Tables 1 and 7, and hence the more negative output gaps of Table 7 result both from the different assessment of the economy's cyclical position in 1999/2000 and from the lower expected GDP growth in 2002/2003.

Table 8 – Results from the CES production function (23)

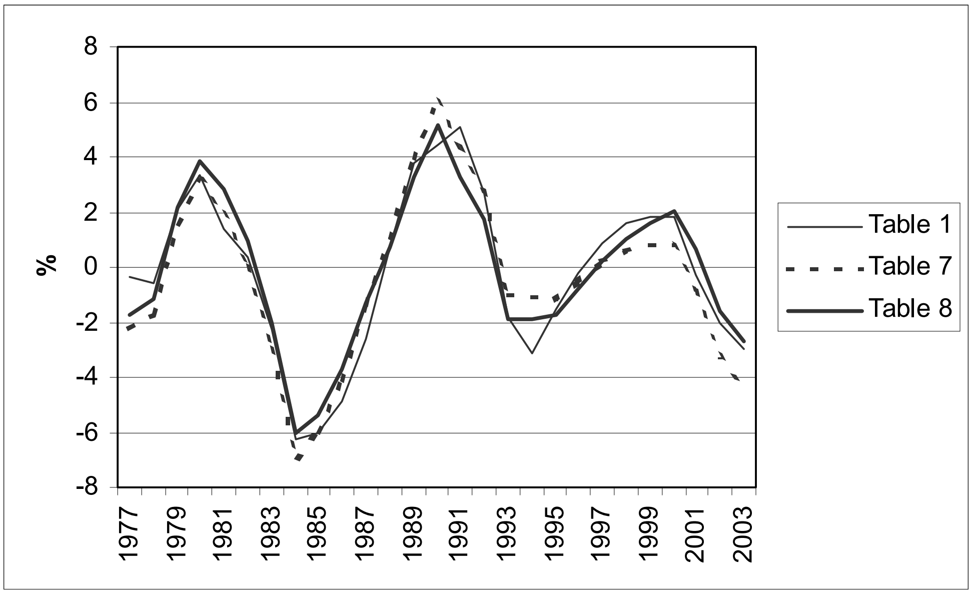
	WY	WY*	WY*S	WY*K	WY*L	YGAP	TGAP	PGAP	UGAP
1977						-1.7	-1.2	0.3	-0.8
1978	3.4	2.8	1.4	1.9	-0.4	-1.2	-0.1	-0.4	-0.7
1979	6.1	2.6	1.4	1.7	-0.4	2.2	1.5	1.0	-0.3
1980	4.8	3.1	1.4	1.9	-0.1	3.9	3.2	0.4	0.3
1981	1.3	2.3	1.4	2.0	-1.0	2.8	0.4	1.6	0.8
1982	2.1	4.0	1.5	1.7	0.8	0.9	0.7	-0.3	0.5
1983	-0.2	2.8	1.6	1.3	-0.1	-2.0	-1.5	-0.4	-0.1
1984	-1.8	2.4	1.7	0.7	-0.1	-6.0	-4.7	-0.8	-0.5
1985	3.0	2.3	1.8	0.6	-0.1	-5.4	-4.1	-0.4	-0.9
1986	4.1	2.3	2.0	0.6	-0.3	-3.7	-0.6	-2.2	-0.9
1987	5.9	3.4	2.0	0.9	0.4	-1.3	0.6	-1.4	-0.5
1988	5.5	3.4	2.0	1.0	0.3	0.7	1.3	-0.7	0.1
1989	5.4	2.8	2.0	1.0	-0.1	3.3	2.2	0.3	0.7
1990	4.8	2.9	1.9	1.0	-0.1	5.2	2.7	1.4	1.1
1991	2.4	4.3	1.8	0.9	1.5	3.3	0.3	2.0	1.0
1992	1.9	3.5	1.7	0.9	0.8	1.7	0.7	0.4	0.6
1993	-1.4	2.3	1.6	0.7	-0.1	-1.9	-1.4	-0.4	-0.1
1994	2.5	2.4	1.6	0.7	0.2	-1.9	-0.4	-0.7	-0.7
1995	2.9	2.8	1.5	0.6	0.6	-1.7	0.5	-1.1	-1.1
1996	3.5	2.5	1.5	0.7	0.4	-0.8	0.6	-0.3	-1.1
1997	3.9	2.9	1.4	1.0	0.5	0.2	0.9	-0.1	-0.6
1998	4.5	3.8	1.4	1.1	1.3	1.0	1.2	-0.3	0.1
1999	3.5	2.9	1.4	1.1	0.5	1.6	0.9	-0.1	0.8
2000	3.5	3.1	1.4	1.1	0.6	2.0	0.5	0.4	1.1
2001	1.9	3.3	1.5	0.9	0.9	0.7	-1.2	0.9	1.0
2002	1.0	3.3	1.6	0.8	0.9	-1.6	-3.0	0.9	0.5
2003	1.7	3.0	1.7	0.8	0.4	-2.7	-3.4	0.8	-0.2

See the notes under Table 1.

on both sides of equation (23) and differentiating.

[24] Interestingly, Botas *et al.* (1998) and Pinheiro (1998) also argue that the Portuguese output gap in the late nineties was only marginally positive, if at all.

Chart 1 – A Comparison of Output Gap Estimates



Growth accounting in the CES framework points to technical progress, rather than factor accumulation, as the main driving force. The strong contrast with the EC results is partly a question of data, and not only of methodology: as we have seen, Table 2 bridges some of the gap between Table 1 and Tables 7 and 8. Methodological differences also matter, and the fact that we have calibrated the labour share at a higher value than the Commission (0.73 against 0.65), together with Harrod-neutrality, also gives a bigger weight to technical progress[25].

4. Concluding remarks

In this study we have discussed in detail the European Commission's recently adopted method of estimating potential output through a production function approach. For the Portuguese economy, resorting to statistical tests in the framework of cointegrated vector autoregressions, we have rejected the restrictions implied by a Cobb-Douglas functional form, and estimated/calibrated a more general CES production function instead. We have also employed a fully up-to-date dataset, with some significant departures from the AMECO series. Both as a consequence of the new data and of the new CES-based methodology, our results give a bigger role to technical progress as a source of economic growth. They also hint at the possibility, as previous studies had done, that output gaps in the late nineties were only marginally positive; and that in

2002 and 2003 the Portuguese economy may face bigger negative gaps than those estimated by the Commission services.

Potential output is an elusive concept, and there are no perfect methodologies to estimate it. Though we believe to have improved on the Commission's method, there are a number of research strands still to be pursued. First, one could try to estimate the CES parameters on the basis of the demands for both capital and labour, instead of the latter alone (e.g. Dimitz, 2001). Doing so would provide a basis to identify different forms of technical progress[26], and bring further information to bear on the estimation of the elasticity of substitution.

It would also be interesting to depart from the standard assumption of a constant rate of technical progress, though a tradeoff may arise between within-sample accuracy and forecasting ability/simplicity. Last but not least, there is need for further work on the NAIRU estimation, preferably in a way that is integrated with the estimation of potential output. Apel and Jansson (1999) have jointly estimated both concepts in the framework of an unobserved components model; introducing production function considerations in such a framework would be a step forward.

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[25] We illustrate this point in the appendix by redoing Table 8 with the EC's labour share of 0.65.

[26] There are nonetheless considerable problems in measuring the user cost of capital, and a loss of degrees of freedom when moving to a higher-dimensional VAR.

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Table A.1 – A comparison of the AMECO and DGEP datasets

	Y			I			K			L			u			POP			u*		
	AMECO	DGEP	% dif.	AMECO	DGEP	% dif.	AMECO	DGEP	% dif.	AMECO	DGEP	% dif.	AMECO	DGEP	dif.	AMECO	DGEP	% dif.	AMECO	DGEP	dif.
1977	47.5	48.2	-1.6	10.9	11.2	-3.0	93.5	95.4	-2.0	3784.0	4495.1	-15.8	7.3	7.5	-0.2	5890.5	5921.4	-0.5	6.2	6.4	-0.2
1978	48.8	49.9	-2.1	11.5	12.0	-3.8	98.2	100.2	-2.0	3772.0	4425.6	-14.8	7.9	8.1	-0.2	5945.5	6004.6	-1.0	6.9	7.0	-0.2
1979	51.6	52.9	-2.6	11.4	11.7	-3.0	102.8	104.9	-2.0	3854.0	4519.0	-14.7	7.9	8.2	-0.3	6014.5	6094.0	-1.3	7.5	7.7	-0.3
1980	53.9	55.5	-2.8	12.4	12.8	-3.1	108.5	110.6	-1.9	3940.0	4505.4	-12.5	7.6	7.7	-0.1	6121.0	6190.0	-1.1	8.0	8.1	-0.2
1981	54.8	56.2	-2.4	13.0	13.5	-3.3	115.2	117.4	-1.9	3918.0	4552.1	-13.9	7.3	8.2	-0.9	6225.0	6260.2	-0.6	8.3	9.3	-1.0
1982	56.0	57.4	-2.4	13.3	13.7	-2.7	121.8	124.1	-1.9	3928.0	4465.7	-12.0	7.2	7.4	-0.2	6292.1	6324.0	-0.5	8.3	8.0	0.3
1983	55.9	57.2	-2.4	12.4	12.8	-3.0	127.6	130.0	-1.9	4128.0	4414.4	-6.5	8.2	7.9	0.3	6353.6	6383.2	-0.5	8.8	7.8	1.0
1984	54.8	56.2	-2.4	10.2	10.6	-3.4	131.1	133.5	-1.8	4075.0	4347.3	-6.3	8.9	8.5	0.4	6412.4	6441.6	-0.5	8.8	7.7	1.1
1985	56.4	57.9	-2.6	9.9	10.2	-3.6	133.9	136.3	-1.8	4057.0	4346.5	-6.7	9.1	8.6	0.5	6457.0	6472.4	-0.2	8.3	7.4	0.9
1986	58.7	60.3	-2.6	10.9	11.3	-3.6	137.1	139.5	-1.8	4059.7	4228.7	-4.0	8.8	8.5	0.3	6484.9	6497.5	-0.2	7.7	7.3	0.4
1987	62.5	63.9	-2.2	12.9	13.3	-2.7	141.8	144.2	-1.7	4148.2	4325.6	-4.1	7.2	7.1	0.1	6506.4	6515.3	-0.1	6.4	6.4	-0.1
1988	67.1	67.4	-0.4	14.8	14.8	0.2	147.7	149.7	-1.3	4252.4	4422.7	-3.9	5.8	5.7	0.1	6522.7	6530.3	-0.1	5.6	5.8	-0.2
1989	71.5	71.1	0.6	15.4	15.4	-0.3	153.9	155.4	-1.0	4346.7	4506.7	-3.6	5.2	5.0	0.2	6535.9	6542.1	-0.1	5.7	5.9	-0.2
1990	74.3	74.4	-0.2	16.5	16.7	-0.9	160.5	161.7	-0.7	4438.5	4583.9	-3.2	4.8	4.7	0.1	6548.7	6558.9	-0.2	5.8	6.0	-0.3
1991	77.5	76.3	1.7	17.1	17.3	-1.0	167.2	167.9	-0.4	4562.7	4712.0	-3.2	4.2	4.1	0.1	6570.0	6648.3	-1.2	5.2	5.4	-0.2
1992	78.4	77.7	0.9	17.9	18.1	-1.3	173.9	174.4	-0.2	4468.4	4635.0	-3.6	4.3	4.1	0.2	6602.5	6681.8	-1.2	4.9	4.9	0.0
1993	76.8	76.6	0.2	16.9	17.0	-1.0	179.4	179.4	0.0	4389.0	4540.2	-3.3	5.6	5.5	0.1	6639.7	6713.9	-1.1	5.5	5.4	0.1
1994	77.5	78.5	-1.3	17.3	17.6	-1.8	184.9	184.6	0.2	4381.6	4492.8	-2.5	6.9	6.8	0.1	6679.9	6753.9	-1.1	6.1	5.8	0.2
1995	80.8	80.8	0.0	18.5	18.5	0.0	190.6	189.6	0.5	4358.4	4483.7	-2.8	7.3	7.2	0.1	6708.7	6788.8	-1.2	6.2	5.8	0.4
1996	83.9	83.7	0.3	19.6	19.5	0.6	197.0	195.3	0.9	4388.4	4554.7	-3.7	7.3	7.3	0.0	6729.7	6817.9	-1.3	6.4	6.0	0.5
1997	87.2	87.0	0.3	22.3	22.2	0.6	205.6	203.2	1.2	4477.3	4626.2	-3.2	6.8	6.7	0.1	6750.3	6848.4	-1.4	6.5	6.0	0.6
1998	91.2	91.0	0.2	24.8	24.8	0.3	216.2	212.9	1.5	4597.4	4732.6	-2.9	5.1	5.0	0.1	6769.8	6879.6	-1.6	5.6	5.2	0.4
1999	94.3	94.2	0.1	26.6	26.6	0.2	227.7	223.3	2.0	4681.2	4817.8	-2.8	4.5	4.4	0.1	6784.4	6911.3	-1.8	5.5	5.4	0.1
2000	97.5	97.5	0.0	27.9	27.8	0.3	240.4	234.2	2.6	4762.9	4901.0	-2.8	4.1	4.0	0.1	6801.6	6940.7	-2.0	5.1	5.3	-0.3
2001	99.2	99.3	-0.1	27.8	27.9	-0.6	252.1	244.6	3.1	4842.0	4981.4	-2.8	4.1	4.1	0.0	6890.1	6994.1	-1.5	4.6	5.3	-0.7
2002	100.6	100.3	0.3	27.9	27.9	0.2	263.2	254.1	3.6	4854.2	5005.1	-3.0	4.6	4.5	0.1	6938.3	7044.3	-1.5	4.5	5.1	-0.7
2003	102.9	102.1	0.7	28.5	28.6	-0.5	274.1	263.6	4.0	4871.4	4984.6	-2.3	5.0	5.2	-0.2	6993.8	7061.7	-1.0	4.3	5.0	-0.7

The AMECO series are those mentioned in footnote 3, and DGEP's are described in section 3.1. Units are billion (10^9) euros for *Y*, *I* and *K*, and thousand people for *L* and *POP*. Employment (*L*) comprises both employees and the self-employed. The final columns present the NAIRU series estimated by the methodology of the Commission services (see section 2.2), applied both to AMECO and to DGEP data.

Table A.2 - Results from the CES production function (23)

	WY	WY*	WY*S	WY*K	WY*L	YGAP	TGAP	PGAP	UGAP
1977						-1.5	-1.1	0.3	-0.7
1978	3.4	2.9	0.9	2.4	-0.3	-1.1	-0.1	-0.3	-0.6
1979	6.1	2.7	1.0	2.1	-0.3	2.3	1.7	0.8	-0.3
1980	4.8	3.2	1.0	2.3	-0.1	3.9	3.3	0.3	0.3
1981	1.3	2.5	1.0	2.5	-0.9	2.6	0.5	1.4	0.7
1982	2.1	4.0	1.1	2.2	0.7	0.7	0.5	-0.2	0.5
1983	-0.2	2.8	1.2	1.7	-0.1	-2.2	-1.8	-0.4	-0.1
1984	-1.8	2.2	1.4	0.9	-0.1	-6.1	-5.0	-0.7	-0.5
1985	3.0	2.2	1.5	0.7	-0.1	-5.4	-4.3	-0.4	-0.8
1986	4.1	2.3	1.7	0.8	-0.2	-3.7	-0.9	-2.0	-0.8
1987	5.9	3.3	1.8	1.2	0.4	-1.2	0.5	-1.3	-0.5
1988	5.5	3.4	1.8	1.3	0.3	0.8	1.3	-0.6	0.1
1989	5.4	2.9	1.8	1.3	-0.1	3.3	2.3	0.3	0.7
1990	4.8	3.0	1.7	1.3	-0.1	5.1	2.9	1.2	1.0
1991	2.4	4.2	1.6	1.2	1.3	3.3	0.6	1.8	0.9
1992	1.9	3.4	1.5	1.2	0.7	1.8	0.8	0.4	0.6
1993	-1.4	2.3	1.4	0.9	0.0	-1.9	-1.4	-0.4	-0.1
1994	2.5	2.4	1.4	0.9	0.1	-1.9	-0.5	-0.6	-0.7
1995	2.9	2.7	1.3	0.8	0.5	-1.6	0.4	-1.0	-1.0
1996	3.5	2.5	1.2	0.9	0.3	-0.6	0.6	-0.2	-1.0
1997	3.9	3.0	1.2	1.3	0.5	0.3	1.0	-0.1	-0.5
1998	4.5	3.8	1.2	1.5	1.1	1.1	1.2	-0.3	0.1
1999	3.5	3.1	1.2	1.5	0.4	1.5	0.9	-0.1	0.7
2000	3.5	3.2	1.2	1.4	0.5	1.8	0.5	0.4	1.0
2001	1.9	3.4	1.3	1.2	0.8	0.4	-1.4	0.9	0.9
2002	1.0	3.4	1.5	1.1	0.8	-1.9	-3.2	0.8	0.4
2003	1.7	3.1	1.6	1.0	0.4	-3.2	-3.8	0.8	-0.2

Same as Table 8, but with a 0.65 labour share in income.